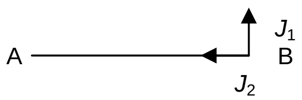
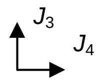
1(i)	$m = \frac{4}{3}\pi r^3 \rho$	M1	Expression for *m*	
	$\frac{dm}{dt} = 4\pi r^2 \rho \frac{dr}{dt}$	M1	Relate $\frac{dm}{dt}$ to $\frac{dr}{dt}$	
	$\lambda \cdot 4\pi r^2 = 4\pi r^2 \rho \frac{dr}{dt}$	M1	Use of $\frac{dm}{dt}$ proportional to surface area	
	$\frac{dr}{dt} = \frac{\lambda}{\rho} = k$	E1	Accept alternative symbol for constant if used correctly (here and subsequently)	
	$r = r_0 + kt$	M1	Integrate and use condition	
	$m = \frac{4}{3}\pi\rho(r_0 + kt)^3$	A1		
				6
(ii)	$\frac{d}{dt}(mv) = mg$	M1	N2L	
	$mv = \int mg \ dt = \int \frac{4}{3}\pi\rho(r_0+kt)^3 g \ dt$	M1	Express *mv* as an integral	
	$= \frac{4}{3}\pi\rho g\left[\frac{1}{4k}(r_0+kt)^4 + c\right]$	M1	Integrate	
	$t = 0, v = 0 \Rightarrow c = -\frac{1}{4k}{r_0}^4$	M1	Use condition	
	$\frac{4}{3}\pi\rho(r_0+kt)^3 v = \frac{4}{3}\pi\rho g \cdot \frac{1}{4k}\left[(r_0+kt)^4 - {r_0}^4\right]$	M1	Substitute for *m*	
	$v = \frac{g}{4k}\left[r_0 + kt - \frac{{r_0}^4}{(r_0+kt)^3}\right]$	A1		
				6
2(i)	$AP = 2a\cos\theta$	M1	Attempt AP in terms of θ	
	$PB = \frac{5}{2}a - 2a\cos\theta$	E1		
	$V = -mg \cdot PB - mg \cdot PA\cos\theta$	M1	Attempt *V* in terms of θ	
	$= -mg\left(\frac{5}{2}a - 2a\cos\theta\right) - mg\left(2a\cos\theta\right)\cos\theta$			
	$= -mga\left(2\cos^2\theta - 2\cos\theta + \frac{5}{2}\right)$	E1		
				4
(ii)	$\frac{dV}{d\theta} = mga\sin\theta\left(4\cos\theta - 2\right)$	M1	Differentiate	
	$\frac{dV}{d\theta} = 0 \Rightarrow \sin\theta = 0 \text{ or } \cos\theta = \frac{1}{2}$	M1	Solve	
	$\Rightarrow \theta = 0 \text{ or } \pm\frac{1}{3}\pi$	A1	For 0 and either of $\frac{1}{3}\pi$ or $-\frac{1}{3}\pi$	
	$\frac{d^2V}{d\theta^2} = mga\sin\theta\left(-4\sin\theta\right) + mga\cos\theta\left(4\cos\theta - 2\right)$	M1 A1	Differentiate again	
	$\theta = 0 \Rightarrow \frac{d^2V}{d\theta^2} = 2mga > 0 \Rightarrow$ stable	M1 F1	Consider sign of V'' in one case Correct deduction for one value of θ	
	$\theta = \pm\frac{1}{3}\pi \Rightarrow \frac{d^2V}{d\theta^2} = -3mga < 0 \Rightarrow$ unstable	F1	Correct deduction for another value of θ N.B. Each F mark is dependent on both M marks. To get both F marks, the two values of θ must be physically possible (i.e. in the first or fourth quadrant) and not be equivalent or symmetrical positions.	
				8

3(i)	$P = Fv = mv\dfrac{dv}{dx}v$	M1	Use of $P = Fv$	
	$v^2\dfrac{dv}{dx} = 0.0004\left(10\,000v + v^3\right)$	A1	Or equivalent	
	$\int \dfrac{v}{10\,000+v^2}\,dv = \int 0.0004\,dx$	M1	Separate variables	
	$\frac{1}{2}\ln\left\|10\,000+v^2\right\| = 0.0004x + c$	M1	Integrate	
	$v^2 = Ae^{0.0008x} - 10\,000$	M1	Rearrange	
	$x = 0, v = 0 \Rightarrow A = 10\,000$	M1	Use condition	
	$v = 100\sqrt{e^{0.0008x} - 1}$	A1		
	$x = 900 \Rightarrow v = 102.7 > 80$ so successful			
	or $v = 80 \Rightarrow x = 618.37 < 900$ so successful	E1	Show that their *v* implies successful take off	
				8
(ii)	$v\dfrac{dv}{dt} = 0.0004\left(10\,000v + v^3\right)$	F1	Follow previous DE	
	$\int \dfrac{1}{10\,000+v^2}\,dv = \int 0.0004\,dt$	M1	Separate variables	
	$\frac{1}{100}\tan^{-1}\left(\frac{1}{100}v\right) = 0.0004t + k$	M1	Integrate	
		A1		
	$t = 0, v = 0 \Rightarrow k = 0$	M1	Use condition	
	$\Rightarrow v = 100\tan(0.04t)$	E1	Clearly shown	
	$v \to \infty$ at finite time suggests model invalid	B1		
				7
(iii)	$t = 11 \Rightarrow v = 47.0781$	B1	At least 3sf	
	Hence maximum $P = 230.049m$	M1	Attempt to calculate maximum *P*	
	$v = 47.0781 \Rightarrow x = 250.237$	M1	Use solution in (i) to calculate *x*	
	$v^2\dfrac{dv}{dx} = 230.049$	M1	Set up DE for $t \geqslant 11$. Constant acceleration formulae $\Rightarrow$ M0.	
	$\frac{1}{3}v^3 = 230.049x + B$	M1	Separate variables and integrate	
		F1	Follow their maximum *P* (condone no constant)	
	$v = 47.0781, x = 250.237 \Rightarrow B = -22786.3$	M1	Use condition on *x*, *v* (not $v = 0$, not $x = 0$ unless clearly compensated for when making conclusion). Constant acceleration formulae $\Rightarrow$ M0.	
	$v = 80 \Rightarrow x = 840.922$ or $x = 900 \Rightarrow v = 82.0696$	M1	Relevant calculation. Must follow solving a DE.	
	so successful	A1	All correct (accept 2sf or more)	
				9

4(i)	Considering elements of length $\delta x \Rightarrow I = \int_0^{2a} \rho x^2 \, dx$	M1	Set up integral	
	$= \frac{M}{8a^2}\int_0^{2a}\left(5ax^2 - x^3\right)dx$	M1	Substitute for ρ in predominantly correct integral	
	$= \frac{M}{8a^2}\left[\frac{5}{3}ax^3 - \frac{1}{4}x^4\right]_0^{2a}$	M1	Integrate	
	$= \frac{7}{6}Ma^2$	E1		
	Considering elements of length $\delta x \Rightarrow M\bar{x} = \int_0^{2a} \rho x \, dx$	M1	Set up integral	
	$= \frac{M}{8a^2}\int_0^{2a}\left(5ax - x^2\right)dx$	M1	Substitute for ρ in predominantly correct integral	
	$= \frac{M}{8a^2}\left[\frac{5}{2}ax^2 - \frac{1}{3}x^3\right]_0^{2a}$	M1	Integrate	
	$\bar{x} = \frac{11}{12}a$	E1		
				8
(ii)	$\frac{1}{2}I\dot{\theta}^2 = Mg \cdot \frac{11}{12}a(1-\cos\theta)$	M1	KE term in terms of angular velocity	
		B1	$\pm Mg \cdot \frac{11}{12}a\cos\theta$ seen	
		M1	energy equation	
	$\dot{\theta} = \sqrt{\frac{11g}{7a}(1-\cos\theta)}$	A1		
				4
(iii)	 $\theta = \frac{1}{2}\pi \Rightarrow \dot{\theta} = \sqrt{\frac{11g}{7a}}$	F1	Their $\dot{\theta}$ at $\theta = \frac{1}{2}\pi$	
	$2a \cdot (-J_1) = I\left(0 - \sqrt{\frac{11g}{7a}}\right)$	M1	Use of angular momentum	
		A1	Correct equation (their $\dot{\theta}$)	
	$J_1 = \frac{1}{12}M\sqrt{77ag}$	E1		
	$J_2 = \frac{1}{12}M\sqrt{77ag}$	B1	Correct answer or follow their J_1	
				5
(iv)	 $J_4 = J_2$	M1	Consider horizontal impulses	
	$= \frac{1}{12}M\sqrt{77ag}$	F1	Follow their J_2	
	$J_3 + J_1 = M \cdot \frac{11}{12}a\sqrt{\frac{11g}{7a}}$	M1	Vertical impulse-momentum equation	
		M1	Use of $r\dot{\theta}$	
	$J_3 = \frac{1}{21}M\sqrt{77ag}$	A1	cao	
	angle $= \tan^{-1}\left(\frac{J_3}{J_4}\right) = \tan^{-1}\left(\frac{\frac{1}{21}M\sqrt{77ag}}{\frac{1}{12}M\sqrt{77ag}}\right)$	M1	Must substitute	
	$= \tan^{-1}\left(\frac{4}{7}\right) \approx 0.519\,\text{rad} \approx 29.7°$	A1	cao (any correct form)	
				7